

QUIZ 1 TOPICS:

- * Fallacies (§2A)
- * Logical Operators (§2B)
- * Truth Tables (§2B)
- * Propositions
- * Conditional, converse, inverse, contrapositive
- * Sets of numbers, recognizing irrational and rational numbers, and using set notation

§3A Percentages

Review of Percentages:

Per cent means "per 100" or "out of 100."

Per cent is often written as %.

$$\text{Ex: } 50\% = \frac{50}{100} = \frac{1}{2} = .5$$

$$\text{Ex: } 25\% = \frac{25}{100} = \frac{1}{4} = .25$$

$$\text{Ex: } 157\% = \frac{157}{100} = 1.57$$

$$\text{Ex: } .05\% = \frac{.05}{100} = \frac{5}{10000} = .0005$$

$$\text{Ex: } 100\% = \frac{100}{100} = 1$$

Convert decimal to percent: Move decimal to the right two places.

$$\text{Ex: } .15 = 15\%$$

Convert fraction to percent: Convert fraction to decimal and then convert as above.

Convert a percent to a fraction: Replace % with division by 100 and simplify if needed.

$$\text{Ex: } 101\% = \frac{101}{100}$$

$$\text{Ex: } -5\% = -\frac{5}{100} = -\frac{1}{20}$$

Convert percentage to decimal: Drop the % and shift decimal two places to left.

$$\text{Ex: } 23\% = 0.23$$

$$\text{Ex: } 1050\% = 10.5$$

Uses of percentages:

1) Percents can be used to represent fractions.

Ex: 20% of the vehicles on a car lot were damaged by hail. How many cars were damaged if there were 200 cars in total on the lot?

$$\text{SOLUTION: } 20\% = \frac{20}{100} = \frac{1}{5}$$

$$\times 200 \text{ cars} \cdot \frac{1}{5} = 40 \text{ cars}$$

$$\times 200 \text{ cars} \cdot .2 = 40 \text{ cars}$$

Ex: 500 people in the organization, 25% of the total organization, own two cars. How many people are in the organization?

$$\text{SOLUTION: } 25\% = \frac{1}{4}$$

Since 500 people represent 25% of the organization, they account for $\frac{1}{4}$ of it, or

$$500 \text{ people} \times 4 = 2000 \text{ people}$$

There are 2000 people in the organization.

2) Change can be explained using percents.

Absolute change = New value - original value

Ex: A computer costs 800 USD today.

One month ago, it cost 1,000 USD.

What is the Absolute price change?

Solution: $A = 800 \text{ USD} - 1000 \text{ USD}$

$$= -200 \text{ USD}$$

The cost dropped by 200 USD.

Relative change = $\frac{\text{Absolute change}}{\text{original value}}$

Relative change can be written as a percent by multiplying by 100%

Ex: what is the relative price change?

Solution: $R = \frac{-200 \text{ USD}}{1000 \text{ USD}} = -\frac{1}{5} = -.2$

As a percent: $-.2 \times 100\% = -20\%$

Ex: The population of a city grew from 1,000 people in 1970 to 3,500 people in 1982. What was the absolute and relative change in population from 1970 to 1982?

SOLUTION:

$$A = 3,500 \text{ people} - 1,000 \text{ people} \\ = 2,500 \text{ people}$$

$$R = \frac{2,500 \text{ people}}{1,000 \text{ people}} = \frac{5}{2} = 2.5 \\ = 2.5 \times 100\% = 250\%$$

"The new population is 250% more than the original."

Ex: The number of violent crimes in city X decreased over 5 years from 150 in 1995 to 45 in 2001. What are the relative and absolute changes?

Solution:

$$A = 45 \text{ crimes} - 150 \text{ crimes} \\ = -105 \text{ crimes}$$

$$R = \frac{-105 \text{ crimes}}{150 \text{ crimes}} = -.7 = -70\%$$

"The new crime rate is 70% less than the original."

3) Percents are also used for comparison.

Absolute difference = compared value

Relative difference = $\frac{\text{Absolute difference} - \text{reference value}}{\text{reference value}}$

Ex: Compare these two cars in terms of cost using both absolute and relative differences:

Lexus at \$40,000

Mercedes at \$50,000

Solution: Comparing the Mercedes to the Lexus:

$$* A = \$50,000 - \$40,000 = \$10,000$$

IE, the Mercedes is \$10,000 more than the Lexus.

Comparing the Lexus to the Mercedes:

$$* A = \$40,000 - \$50,000 = -\$10,000$$

IE, the Lexus is \$10,000 less than the Mercedes.

These are the two absolute differences.

Comparing the Mercedes to the Lexus:

$$R = \frac{\$10,000}{40,000} = 25\%$$

IE, the Mercedes costs 25% more than the Lexus.

Another way is to compare the Lexus to Mercedes, giving a relative difference of:

$$R = \frac{-\$10,000}{\$50,000} = -20\%$$

IE, the Lexus costs 20% less than the Mercedes.

Review of Ratios: A ratio makes a comparison by division. IT HAS NO UNITS.

Ex: What is the ratio of the price of Computer A to computer B if Computer A costs \$600 and Computer B costs \$1200?

SOLUTION:

$$\frac{\text{Computer A cost}}{\text{Computer B cost}} = \frac{\$600}{\$1200} = \frac{1}{2} \\ = .5 = 50\%$$

Ex: A is the 8 million population of NC and B is the 24 million population of TX. State two ratios.

Solution

A to B : 8 to 24 : 1 to 3

B to A : 24 to 8 : 3 to 1

A is 33% of B.

Another example of Percentage difference:

Ex: In 2004, France was the number 1 tourist destination with 77 million international arrivals. Spain ranked second with 50 million. Use Spain as the reference value.

SOLUTION:

Absolute difference:

$$77 \text{ million} - 50 \text{ million} = 27 \text{ million}$$

Relative difference:

$$\frac{27 \text{ million}}{50 \text{ million}} = .54 = 54\%$$

Percentages of Percentages

Ex: In 25 years, the percentage of the world's population living in developed countries decreased from 37% to 20%. Determine the change in percentage points and as a percent.

SOLUTION:

$$A = 20\% - 37\% = -7\%$$

$$R = \frac{-7\%}{37\%} = -25.9\%$$

Tax Calculations:

Ex: You purchase a \$25.00 item from a mail-order catalog. It says you must include 6% tax. How much tax will you owe? How much does the product cost plus tax?

SOLUTION:

$$\text{Tax} = \$25 \times 6\% = \$25 \times .06 = \$1.50$$

$$\text{Total} = \$25 + \$1.50 = \$26.50$$

Abuses of Percentages:

Impossibilities:

Ex: A golf course advertises that its membership costs have fallen 200%.

Does this make sense? If the cost was originally \$100, what is the new cost?

SOLUTION: This makes no sense. Here's why: The cost will drop by 200% of \$100, or

$$\$100 \times 2 = \$200$$

So, the new cost is

$$\text{New cost} = \$100 - \$200 = -\$100$$

IE, the golf course pays you \$100 to join.